Objectives:

- Gain practises with the stabilizer formalism by deriving some properties of two example codes.

The five-qubit code is defined via the following set of independent stabilizer generators

\[
\begin{align*}
XZXXI \\
IXZXX \\
XIXZZ \\
ZXIXZ
\end{align*}
\]

1. How many logical qubits are encoded in this code?

2. We usually define a logical encoded \( Z \) operator as \( \tilde{Z} = ZZZZ \). Verify that this operator commutes with all generators of the stabilizer. Propose an encoded logical \( X \) operator \( \tilde{X} \) which commutes with all stabilizer generators, but anti-commutes

3. Find the minimal weight encoded logical Pauli operator and hence the distance of this code.

Hint: I do not recommend that you do this by computing the full centralizer of the code (it has 64 elements!). Instead consider how Pauli operators multiply together, and under what circumstances multiplying two operators together reduces its weight.

4. A logical qubit is encoded in the 5-qubit code. The code undergoes an error described by Pauli operator \( YYIII \). The stabilizer generators are now measured. Following the error, what are the outcomes of each measurement?
5. Now consider a different code, defined by the following set of independent stabilizer generators

\[
\begin{align*}
Z & Z Z Z \\
X & X X X X
\end{align*}
\]

How many logical qubits are encoded in this code?

6. Find a set of logical encoded Pauli operators $Z$ and $X$ for each of the encoded qubits in this new code. (The solution to this is not unique, but conventionally, one tries to construct logical $Z$ operators out of physical $Z$ operators and logical $X$ out of $X$ operators).

7. What is the distance of the code you have found? How many single qubit errors can it detect and correct?